1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1,Class XI)
Module Name/Title	Unit 6,Module 3,Gravitational field and Gravitational potential Chapter 8, Gravitation
Module Id	Keph_10803_eContent
Pre-requisites	Acceleration due to gravity g, Variation of g with altitude and depth, Work-Energy theorem, Conservative force
Objectives	After going through this lesson, the learners will be able to: • **Recognize** Gravitational force as a conservative force and hence define gravitational potential energy. • **Understand** Inter conversion of gravitational potential energy and the kinetic energy for the conservation of mechanical energy. • **Conceptualize and define** action by a force at a distance in agravitational field • **Relate** gravitational field and the gravitational potential.
Keywords	Conservative force, Gravitational potential energy, Gravitational field, Gravitational potential

2. Development Team

Role	Name	Affiliation
National MOOC Coordinator (NMC)	Prof. Amarendra P. Behera	Central Institute of Educational Technology, NCERT, New Delhi
Programme Coordinator	Dr. Mohd. Mamur Ali	Central Institute of Educational Technology, NCERT, New Delhi
Course Coordinator / PI	Anuradha Mathur	Central Institute of Educational Technology, NCERT, New Delhi
Subject Matter Expert (SME)	Smita Fangaria	PGT Physics Developer Anveshika Amity International School, Noida
Review Team	Associate Prof. N.K. Sehgal (Retd.) Prof. V. B. Bhatia (Retd.) Prof. B. K. Sharma (Retd.)	Delhi University Delhi University DESM, NCERT, New Delhi

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1. UNIT SYLLABUS

Unit VI: Gravitation

Chapter 8: Gravitation

Kepler's laws of planetary motion, universal law of gravitation.
Acceleration due to gravity and its variation with altitude and depth.

Gravitational potential energy and gravitational potential; escape velocity; orbital velocity of a satellite; Geo-stationary satellites.

2. MODULE WISE DISTRIBUTION OF UNIT SYLLABUS 5 MODULES

The above unit is divided into five modules for better understanding.

Module 1	• Gravitation • Laws of gravitation • Early studies • Kepler's laws
Module 2	• Acceleration due to gravity

	• Variation of g with altitude • Variation of g due to depth • Other factors that change g
Module 3	• Gravitational field • Gravitational energy • Gravitational potential energy • Need to describe these values
Module 4	• Satellites • India's satellite programme and target applications • Geo stationary satellites and Polar satellites • Escape velocity • India's space program
Module 5	• Numerical problems based on Gravitation

MODULE 3

3. WORDS YOU MUST KNOW

- **Gravitational force**: Force of attraction between two objects of some mass.
- **Celestial bodies:** Stars, Planets, comets, asteroids etc.
- **Ellipse**: A regular oval shaped curve which is the locus of a point moving in a plane so that the sum of its distances from two other points (the foci) is constant,
- **Eccentricity of an ellipse**: It is the measure of deviation of the ellipse from circularity.
- **Areal velocity:** It is the rate at which area is swept out by a particle as it moves along a curve. In Kepler's law of areas, the particle is the planet and curve is the orbit in which it moves around the sun.

- **Kepler's lawsof Planetary Motion**

Law of orbits:

All planets move in elliptical orbits with the Sun situated at oneof the foci of the ellipse.

Law of Areas:

The line that joins any planet to the suns weeps equal areas inequal intervals of time

Law of periods:

The Square of the time period of revolution of a planet is proportional to the cube of the semi-major axis of the ellipse traced out by the planet.

- **Newton's Universal Law of gravitation**: It states that the gravitational force between two point masses is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

- **Principle of superposition**: If we have a collection of point masses, the force on any one of them is the vector sum of the gravitational forces exerted by the other point masses
- **Universal gravitational constant**: Denoted by the letter G, it is an empirical physical constant involved in the calculation of gravitational effects in Newton's law of universal gravitation. It is a universal constant with the value 6.673×10^{-11} N m^2 kg^{-2}.

- **Acceleration due to gravity**: Acceleration experienced by an object due to the force of gravitational attraction of the earth.

- **Variation of acceleration due to gravity**: The acceleration due to gravitydecreases with both altitude and depth.
- **Work Energy theorem**: The change in kinetic energy of the object is equal to the net work done by all the forces acting on the object.
- **Conservative force**: The work done by this force is independent of the path taken and in a round trip; the work done by it is zero.
- **Mechanical energy**: It is the sum of kinetic energy and potential energy.

4. INTRODUCTION

In your earlier modules on mechanical force and mechanical work, you will recall considering a force to be conservative if the work done by it on any object did not depend on the total path length covered by the object rather it depended only on the initial and final position of the object.

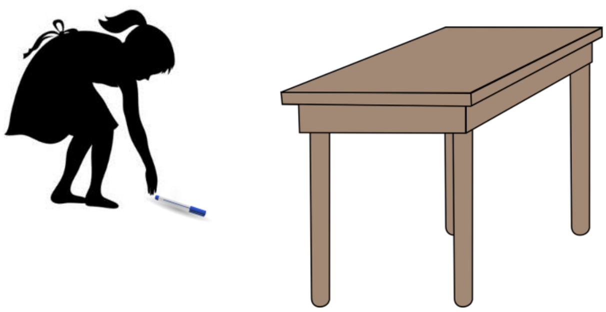
Let us study this with a few examples.

GRAVITATIONAL FORCE IS CONSERVATIVE

Gravitational force on an object is the force by which the object is pulled towards the centre of the earth. For small heights from the ground this force is nearly constant and is equal to mg, where m is the mass of the object and g is the acceleration due to gravity. This force of gravity on the object is considered to be constant because the value of g varies negligibly for small heights.

Now let us consider the following situation:

A teacher while teaching in the class accidentally drops her pen, which was lying on the table, on to the ground. The height of the table is 1m and the mass of the pen is 50gram.



The **possible reaction of three students is quoted below:**

1. **Student A picks up the pen from the ground and keeps it back on the table.**
2. **Student B picks up the pen from the ground, starts writing with it and keeps it back on the table at the end of the class.**
3. **Student C picks up the pen from the ground and puts it in his pocket. He then goes out of the class to drink water. He puts the pen back on the table after coming back to the class.**

In all the situations quoted above, the pen was at rest initially on the ground and finally again it was kept at rest on the table. So the change in the kinetic energy of the pen was zero. Hence the net work done by all the forces acting on the pen is zero.

$\Delta K = W_{net} = 0$

The forces acting on the pen when it is picked up from the ground and kept on the table are:

A. The force applied by the student and
B. The force of gravity.

For the net work done to be equal to zero, the work done by both these forces must be equal in magnitude and opposite in sign.

Work done by the gravitational force = $\vec{F} \cdot \vec{d}$

$= (mg)(h)\cos 180^o$

$= -0.05 \times 9.8 \times 1$

$= -\ 0.49N$

But we have $W_{net} = W_{gravity} + W_{applied} = 0$

$\therefore W_{applied} = -W_{gravity} = 0.49N$

So **we see that the work done by the gravitational force is the same in all the cases**. Hence the students A, B and C also did the same amount of work in picking up the pen and placing it back on the table. This is irrespective of the fact that the path moved by the pen in all the three cases is different.

We conclude from here that:

- **The work done by the force of gravity and the work done against the force of gravity are equal and opposite.**
- **This work done depends only on the initial and final point.**
- **It is independent of the path taken**.

Hence,**we say that the gravitational force is a conservative force.**

But the students in above situation did some work against the force of gravity on the pen in picking it up and putting it on the table. Where did this energy spent by the student go? We know by the law of conservation of energy that energy can neither be created nor destroyed.

The energy spent by the students in lifting it against the force of gravity was stored in the pen and earth system as the change in the gravitational potential energy of the system

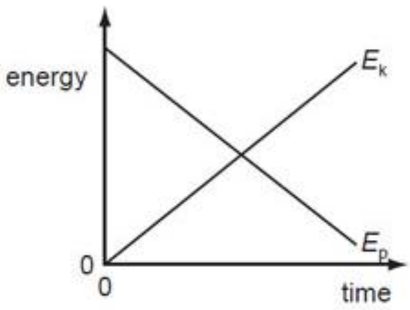
$W_{applied} = -W_{gravity}$ = Change in the gravitational potential energy = ΔU

We can define the gravitational potential energy since gravitational force is a conservative force.

2. GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy is associated with the state of separation between objects which attract one another with gravitational forces. Generally we consider some object and the earth in the system and the separation between the two is the distance between the centre of the earth and the object. The object is taken to be a point object in comparison to the size of the earth.

If the separation between the object and earth increases, the potential energy of the earth and the object system increases.



(Graph shows the variation of potential energy E_p versus time and kinetic energy $E_k$versus time for a free falling object from small height above the earth's surface)

If the potential energy of the system increases, the kinetic energy decreases and vice-versa.This interchange of energy from kinetic to potential and vice-versa is in accordance with the law of conservation of mechanical energy. Here the assumption is that there are no dissipative forces.

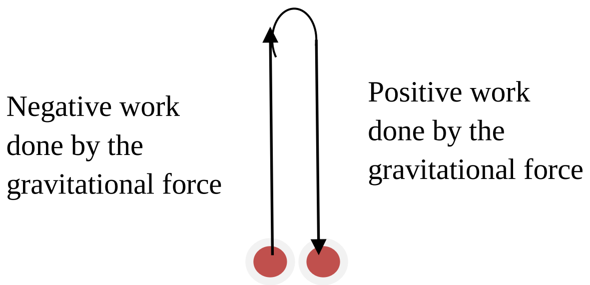
If K represents the kinetic energy and U represents the potential energy, the total mechanical energy conserved can be represented as:

K + U = constant

$\Delta(K+U) = 0$

$\Delta K = -\Delta U$

For example if a ball is thrown up, the potential energy of the ball and earth system increases. The work done by the gravitational force is negative when the ball rises. The speed of the ball decreases which means its kinetic energy decreases during the ascent of the ball. The kinetic energy becomes zero at the top and the potential energy becomes the maximum. In its journey back the reverse process occurs, work done by gravity is positive so the potential energy of the ball decreases while the kinetic energy of the ball increases and becomes maximum just before the ball hits the ground, as shown in figure:



EXAMPLE:

An object of 1kg mass is dropped from a height of 100m. If the dissipative forces due to air friction are neglected we can find the changes in kinetic and potential energy of the object at various points of its journey and justify the result of conservation of mechanical energy.

Potential energy of the object at height 100m = 980J

Kinetic energy of the object at height 100m = 0

Height of the object above the earth's surface (m)	Kinetic energy K of the object (J)	Potential energy U of the object (J)	Changes in the Kinetic energy ΔK of the object(J)	Change in the Potential energy ΔU of the object and earth (J)	Change in the Total energy $(\Delta K + \Delta U)$ of the object. (J)
90	98	882	+ 98	− 98	0

70	294	686	+294	− 294	0
50	490	490	+490	− 490	0
30	686	294	+686	− 686	0
10	882	98	+ 882	− 882	0

Points to be noted:

1. Kinetic energy of the object increases as it falls towards the ground. So ΔK = +ve
2. Potential energy of the object decreases as it falls towards the ground. So ΔU = -ve
3. Increase in the kinetic energy is equal to the decrease in the potential energy.
4. Total mechanical energy is always constant, so the change in the total mechanical energy is always zero.

General expression for the gravitational potential energy:

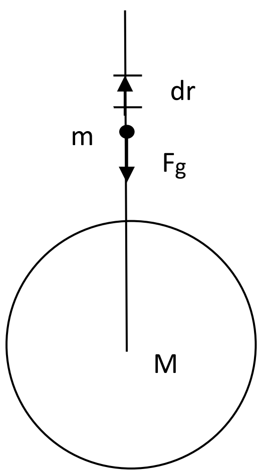
In all the analysis we have done above, we have considered the gravitational force on an object to be constant. But this was only an approximation which was valid for distances close to the earth surface. Close to the earth's surface the variation in the acceleration due to gravity with altitude can be neglected and the distance between the object and the centre of the earth can be taken equal to the radius of the earth. But a general expression for the gravitational potential energy for the earth and the object system will involve actual distances between them.

Suppose a point mass m is moved from its position r_1 to r_2.

This will change the potential energy of the earth mass system

(The position of mass m is with respect to the centre of the Earth having mass M).

The negative of the work done by the gravitational force F_g is equal to the change in this potential energy.



Work done by gravitational force

$=\int_{r_1}^{r_2} \vec{F_g} \cdot \vec{dr} = \int_{r_1}^{r_2} -F_g dr$

$= -\int_{r_1}^{r_2} \frac{GMm}{r^2} dr$

$=\left[\frac{GMm}{r_2} - \frac{GMm}{r_1}\right]$

Change in the potential energy

$=U(r_2) - U(r_1) = \frac{GMm}{r_1} - \frac{GMm}{r_2}$

If $r_1 = \infty$ and $r_2 = r$ which means that the mass m is brought from infinityto the position r.

Change in the potential energy

$=U(r) - U(\infty) = -\frac{GMm}{r}$

The zero of potential energy of the earth and object of mass m system is taken to be at infinity

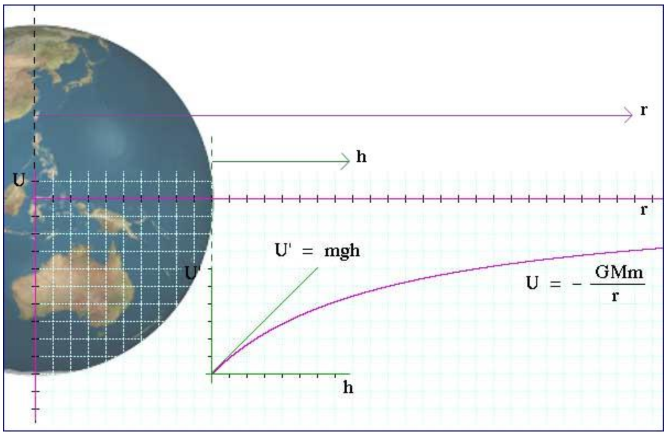
$\therefore U(\infty) = 0$

Hence, $\mathbf{U(r) = -\frac{GMm}{r}}$

This relation is generally referred to as the gravitational potential energy of the object at the position r instead of the potential energy of earth and object system. This is because when the object moves near the earth the changes in potential energy of the earth and object system appears almost entirely as the changes in the kinetic energy of the object itself. Changes in the kinetic energy of the earth are too small to be measured.

This gravitational potential energy is defined as the work done by the external agent in bringing a mass m from infinity to the point r without any acceleration.

Here, the clause of no acceleration is important because it implies no change in the kinetic energy. So all the work done by the external agent only changes the potential energy.



The above graph shows the variation of potential energy with distance from the surface of the earth.

POINTS TO BE NOTED:

- **For distances closer to the surface of the earth the variation in potential energy can be found using the approximate relation U' = mgh**
- **The zero of the potential energy is taken on the surface of the earth in the relation U' = mgh**
- **A more general relation for potential energy is given by U =** $-\frac{GMm}{r}$
- **In this case the zero of the potential energy is taken to be at infinity. This is the maximum possible value of the potential energy.**
- **As the object is brought closer to the earth the potential energy of the object and earth system decreases from its maximum value of zero, hence the value of potential energy is shown as negative.**
- **Close to the surface of the earth the graph for the potential energy shows a linear variation with height but as the distances between the object and the earth increases the graph deviates from linearity. This is when we cannot use the expression U = mgh and hence we see that it is only an approximation valid only very close to the surface of the earth.**

The example given below clarifies this aspect further. Let us find the potential energy of an object of 1kg mass at different heights from the surface of the earth. We will find the potential energy using both the approximate relation (U = mgh) and the general relation

(U= $-\frac{GMm}{r}$)

Potential energy on the surface of the earth using U = mgh = 0

Potential energy on the surface of the earth using U = $-\frac{GMm}{R}$ = $-$ 6.250180 x 10^7 J

Height above the earth surface	U = $-\frac{GMm}{r}$ (x 10^7) J	$\frac{GMm}{r} - \frac{GMm}{R}$	ΔU = mgh (Taking g = 9.8m/s^2)	Difference
100 m	$-$ 6.250082	980 J	980 J	0
1000 m	$-$ 6.249199	9810 J	9800J	10 J
10,000 m	$-$ 6.240385	97950 J	98000 J	50 J
100,000 m	$-$ 6.153592	965880 J	980000 J	14120 J

So the deviation from the actual value increases with distance from the surface of the earth.

At distancesof about 100km from the earth's surface, the approximation

(U = mgh)

is not at all valid.

3. GRAVITATIONAL POTENTIAL ENERGY OF A COLLECTION OF MASSES

Gravitational potential energy of a two particle system separated by a distance r is given by

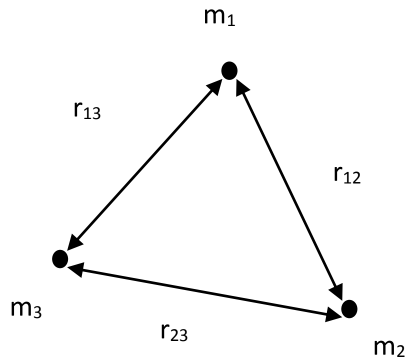
U = $-\frac{Gm_1m_2}{r}$

Note that: Gravitational potential energy is a scalar quantity.

If there are more number of particles the gravitational potential energy of the system of particles is found by taking the gravitational potential energy for each pair of particles into consideration and then algebraically summing up the results.

For example if we consider a three particle system as shown in the figure, the gravitational potential energy of the system is given by

$$U = -\frac{Gm_1m_2}{r_{12}} - \frac{Gm_1m_3}{r_{13}} - \frac{Gm_2m_3}{r_{23}}$$

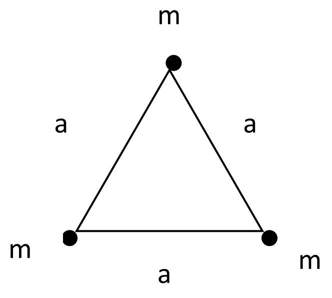


EXAMPLE:

1. **What is the gravitational potential energy of the three particle system having identical mass m which are located at the vertices of an equilateral triangle of side a.**
2. **If you double the side a of the equilateral triangle, what will be the work done**
   i) By you
   ii) By the gravitational force

SOLUTION:

1. Gravitational potential energy $U = -\frac{3Gm^2}{a}$



2 (i) Work done by us (external agent)

W = ΔU = U (final) − U (initial)

$$= \left(-\frac{3Gm^2}{2a}\right) - \left(-\frac{3Gm^2}{a}\right)$$

$$= \frac{3Gm^2}{2a}$$

(ii) Work done by the gravitational force $= -\Delta U = -\frac{3Gm^2}{2a}$

4. GRAVITATIONAL POTENTIAL

If a particle of mass m is moved from point A to point B near the earth, the potential energy of the particle-earth system changes. This change in the potential energy per unit mass of the particle is referred to as the change in the gravitational potential of the particle

$$V_B - V_A = \frac{U_B - U_A}{m}$$

If we move the particle of mass m from infinity (point A) to the point B near the earth, we have $V_A = 0$ which is the reference point having zero potential. V_B is now referred to as the gravitational potential at the point B.

$$V_B = \frac{U_B - U_\infty}{m} = \frac{U_B}{m}$$

This relation holds true because even the potential energy of the earth and the particle system is zero at infinity. Now if the point B is at a distance r from the centre of the earth:

$$U_B = -\frac{GMm}{r}$$

$$\boldsymbol{V_B = -\frac{GM}{r}}$$

POINTS TO BE NOTED:

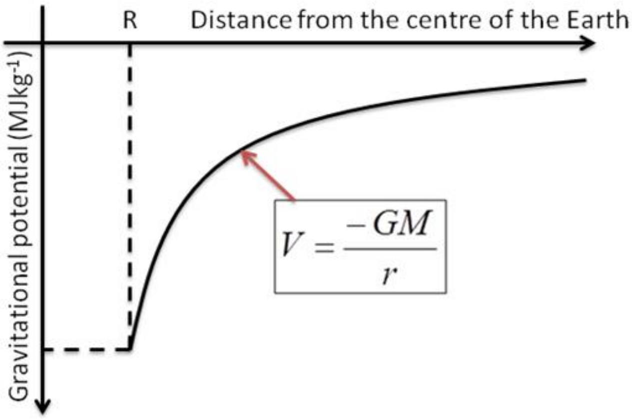
- **V_B, the gravitational potential at the point B varies inversely with distance from the centre of the earth.**
- **The value of gravitational potential is zero at infinity which is its maximum value. As we approach the earth its value decreases from zero and hence becomes negative. This is because gravitational force is an attractive force.**
- **It is independent of the mass of the particle placed at the point B.**
- **The places on or near the earth which have the same gravitational potential are referred to as the equipotential surfaces also known as Geoids.**

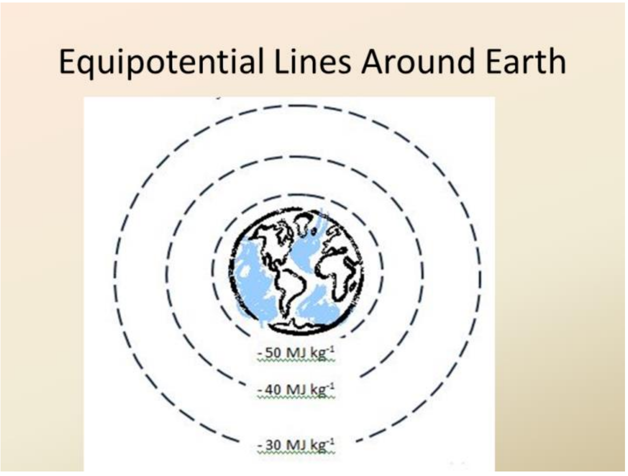
SIGNIFICANCE OF GRAVITATIONAL POTENTIAL:

The gravitational potential can be defined at every point in space around the earth. If the value of the gravitational potential at a point is known we can easily calculate the potential energy of any particle of mass m at that point.

$$U(r) = m\,V(r)$$

This expression relating the gravitational potential and potential energy is true not only for earth – particle system but for any system of two



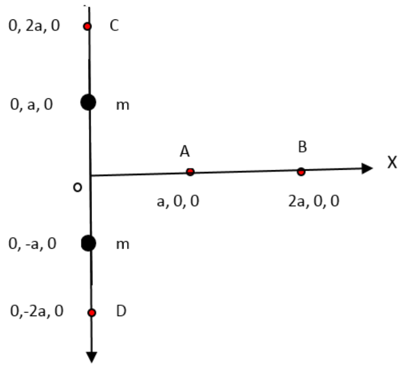


or more particles.

EXAMPLE:

In the figure there are two identical masses m on the y axis located at

(0,a,0) and (0,- a,0)



i) What is the gravitational potential at the points A, B, C and D

ii) What is the work done in taking a M=2kg mass from A to B

iii) What is the work done in takinga M=2kg mass from C to D

SOLUTION

i) $V_A = -\frac{Gm}{a\sqrt{2}} - \frac{Gm}{a\sqrt{2}} = -\frac{2Gm}{a\sqrt{2}}$

$V_B = -\frac{Gm}{a\sqrt{5}} - \frac{Gm}{a\sqrt{5}} = -\frac{2Gm}{a\sqrt{5}}$

$V_C = -\frac{Gm}{a} - \frac{Gm}{3a} = -\frac{4Gm}{3a}$

$V_D = -\frac{Gm}{a} - \frac{Gm}{3a} = -\frac{4Gm}{3a}$

ii) $W_{AB} = M(V_B - V_A) = \frac{4Gm}{a}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right)$

iii) $W_{CD} = M(V_D - V_C) = 0$

Hence, we see that if the potential difference between two points is zero, the work done in moving a mass from one point to the other is zero.

5. GRAVITATIONAL FIELD

Gravitational force acts between particles which are placed at some distance. But how do these particles interact with each other at a distance. To understand this concept of action at a distance, it was considered that the gravitational force exerted by one particle (or a system of particles) on the other particle is a two-step process.

In the first step, the particle of mass m_1 creates a gravitational field in the space around it. This field has an existence of its own. It is defined by a certain magnitude and direction at every point around the mass m_1.

In the second step, when a particle of mass m_2 is brought in the field of m_1, the gravitational field exerts a force on the other particle. This force is proportional to the mass m_2 of the particle brought in the gravitational field ($\vec{E}$) of particle of mass m_1.

$\vec{F} = m_2\vec{E}$

Here $\vec{E}$ is the gravitational field intensity which is a vector quantity. Gravitational field intensity at a point is defined as the gravitational force exerted by the field on a unit mass placed at that point.

6. RELATION BETWEEN GRAVITATIONAL FIELD AND POTENTIAL

Gravitational field and Gravitational potential are both defined in the space around a mass or mass distribution. Gravitational field is a vector quantity which is related to the gravitational acceleration while gravitational potential is a scalar quantity which is related to the work done in moving a unit mass in the gravitational field.

Gravitational field ($\vec{E}$) and potential (V) are related to each other by the following relation

$$E = -\frac{\partial V}{\partial r}$$

If we know the gravitational potential (V), the field intensity can be found by the relation

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

If we know the gravitational field intensity, the potential can be found by the relation:

$$dV = -E_x dx - E_y dy - E_z dz$$

$$\boldsymbol{\Delta V = -\int_{r_1}^{r_2} \vec{E} \cdot \overrightarrow{dr}}$$

EXAMPLE:

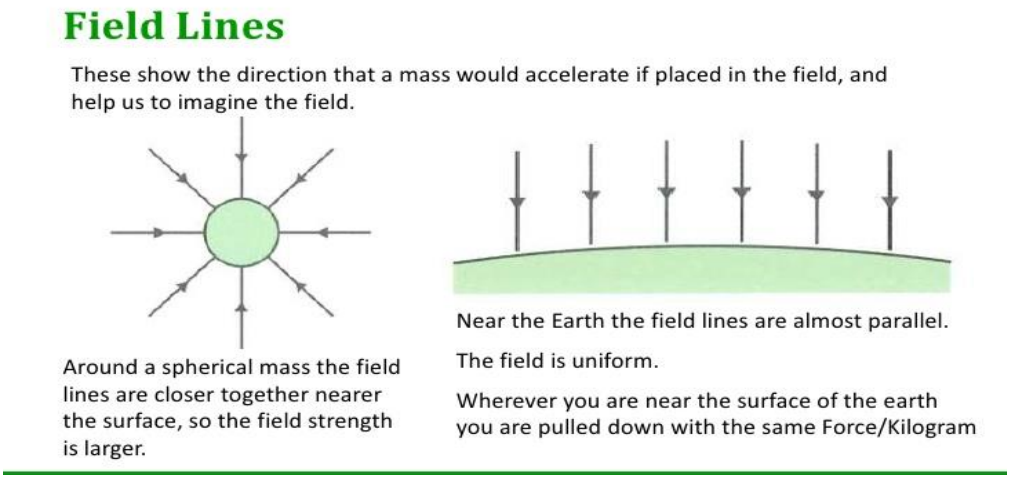
The gravitational field in a region is given by $\vec{E}$= 5$\hat{i}$ + 12$\hat{j}$ N/kg. If the potential at the origin is taken to be zero, find the potential at the points:

- **i) A(12,0)m**
- **ii) B(0, 5) m**
- **iii) C(12,5) m**

SOLUTION:

E_x = 5 N/kg and E_y = 12 N/kg

$$\Delta V = \int_{r_1}^{r_2} \vec{E} \cdot \overrightarrow{dr}$$



In this case $V(0) = 0$

Hence,

i) $V(A) - V(0) = V(A)$

$= V(12,0) = -E_x(x_2 - x_1) - E_y(y_2 - y_1)$

$= -5\,(12 - 0) - 12\,(0) = -60\text{V}$

ii) $V(B) - V(0) = V(B)$

$= V(0, 5) = -E_x(x_2 - x_1) - E_y(y_2 - y_1)$

$= -5\,(0) - 12(5 - 0) = -60\text{V}$

iii) $V(C) - V(0) = V(C)$

$= V(12, 5) = -E_x(x_2 - x_1) - E_y(y_2 - y_1)$

$= -5\,(12-0) - 12\,(5 - 0) = -120\text{V}$

EXAMPLE:

The gravitational potential in a region is given by V = 20(x +y). Find the gravitational field at the point (x,y).

SOLUTION:

$$\vec{E} = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k}$$

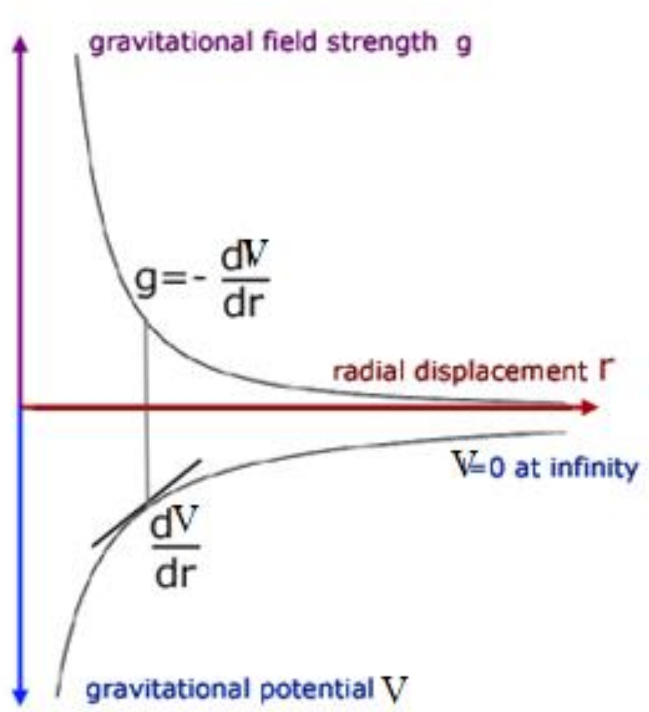
$\vec{E} = -20\hat{\imath} - 20\hat{\jmath}$ N/kg

Gravitational field intensity of the earth is given by:

$\vec{E} = \frac{\vec{F}}{m} = \frac{m\vec{g}}{m} = \vec{g}$

So the intensity of the gravitational field, $\vec{E}$ near the earth's surface is equal to the acceleration due to gravity, $\vec{g}$

The image below shows the pictorial representation of the gravitational field intensity of the earth in the form of the gravitational field lines. The tangent to the field line gives the acceleration due to gravity at that point.



7. SUMMARY

- Gravitational force is a conservative force:

  The work done by the force depends only on the initial and final points and is independent of the path taken.

  The work done by the force in a round trip is zero.

- Gravitational potential energy of a system of two particles is the work done in bringing particle (m_2) from infinity in the gravitational field of the other particle(m_1) which is kept fixed at its position so that the separation between the particles is r.

$$U(r) = -\frac{Gm_1m_2}{r}$$

- Gravitational field intensity is the force experienced in the gravitational field at a point by a unit mass.

$$\vec{E} = \frac{\vec{F}}{m}$$

- Gravitational potential at a point is defined as the Gravitational potential energy per unit mass.

$$V(r) = \frac{U(r)}{m}$$

- Relation between gravitational field and potential

$$E = -\frac{\partial V}{\partial r}$$

- For earth: Gravitational potential on the surface of earth $= -\frac{GM}{R}$, where M is the mass of the earth.
- Gravitational intensity on the surface of earth $\vec{E} = \frac{\vec{F}}{m} = \frac{m\vec{g}}{m} = \vec{g}$

Hence, gravitational field intensity on the surface of the earth is equal to the acceleration due to gravity in magnitude and direction.